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Original Research Article

Influence of Fröhlich electron–phonon interaction on optical parametric processes in polar semiconductor crystals

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ABSTRACT
In polar semiconductor materials, longitudinal optical (LO) phonons generate a macroscopic electric field that interacts strongly with free electrons through the long-range Fröhlich interaction. This electron–phonon coupling results in the formation of a quasiparticle known as a polaron, while its interaction strength is characterized by the dimensionless Fröhlich coupling constant. Because of this strong polar interaction, charge carriers dissipate their excess kinetic energy to the lattice predominantly through LO phonon emission, making it one of the most efficient carrier relaxation mechanisms in III–V bulk semiconductors. In view of the significant role played by the Fröhlich interaction in determining the transport and nonlinear optical properties of semiconductors, it is important to investigate its influence on the performance of optical parametric interactions in compound semiconductor media. In the present work, a theoretical model based on the hydrodynamic model of semiconductor plasma and the coupled-mode theory has been developed to obtain an analytical expression for the threshold pump field required for the onset of the optical parametric process. For numerical analysis, the material parameters of ZnS and GaAs have been employed. These semiconductor crystals are assumed to be irradiated by a 10.6 μm pulsed CO_2 laser, whose optical field is sufficiently intense to induce the required nonlinear response in the medium. The numerical results reveal that semiconductor materials possessing a smaller Fröhlich coupling constant require a lower threshold pump field and therefore provide a more energy-efficient and cost-effective platform for optical parametric interactions. The present analysis offers useful guidelines for the selection of suitable semiconductor materials and is expected to contribute to the design and optimization of nonlinear photonic and optoelectronic devices for advanced industrial and technological applications.

1. Introduction

The optical properties of solid materials are primarily governed by the interaction of incident photons with various elementary excitations, such as electrons, excitons, and phonons. In materials possessing intrinsic structural features, including low-dimensional systems and photonic crystals, the optical response arising from electron and phonon excitations exhibits unique characteristics that have attracted considerable attention for advanced optoelectronic and photonic applications [1]. In particular, polar semiconductor crystals display remarkable optical properties in the infrared (IR) spectral region, where longitudinal and transverse optical phonons become optically active. These phonon modes play a significant role in determining the dielectric response, carrier dynamics, and nonlinear optical behavior of the material, thereby making polar semiconductors promising candidates for infrared photonic devices and nonlinear optical applications.

An optical medium exhibits nonlinear behavior when it is subjected to electromagnetic fields of sufficiently high intensity, causing its polarization to become a nonlinear function of the applied electric field. The nonlinear interaction of high-power, large-amplitude electromagnetic waves with semiconductors, metals, and semimetals has attracted sustained research interest over the past several decades because of its fundamental significance and wide-ranging technological applications [2–7]. These nonlinear interactions form the basis of numerous photonic phenomena, including harmonic generation, optical parametric processes, self-focusing, optical switching, and frequency conversion.

The selection of an appropriate nonlinear medium is a crucial prerequisite for the design and realization of efficient parametric amplifiers, oscillators, and other nonlinear optoelectronic devices. A wide variety of nonlinear materials have been investigated for such applications [8,9]. Among them, semiconductor materials have emerged as particularly attractive due to their exceptional electronic and optical properties, as well as their compatibility with modern device fabrication technologies.

Semiconductors offer several distinct advantages as nonlinear optical media. First, the high concentration of free electrons or holes in doped semiconductors facilitates a wide range of nonlinear optical processes. Second, carrier relaxation times can be tailored through appropriate material engineering and device design, enabling optimization of the nonlinear response. Third, semiconductors exhibit large optical nonlinearities in the vicinity of band-gap resonances, thereby enhancing the efficiency of nonlinear interactions. Fourth, both nonlinear absorption and nonlinear refractive-index changes can be effectively exploited for device operation. Fifth, semiconductor-based nonlinear devices can operate in either normal-incidence or waveguide configurations, providing considerable design flexibility. Furthermore, semiconductor devices can be readily integrated with other optoelectronic components on a single platform, making them highly suitable for photonic integrated circuits. Finally, semiconductors remain substantially transparent for photon energies below their band-gap energies, thereby minimizing linear absorption losses and enabling efficient nonlinear optical interactions [10,11].

These unique characteristics make semiconductor materials one of the most promising platforms for the realization of high-performance nonlinear optical and photonic devices.

In ionic and polar semiconductors, a free charge carrier interacts strongly with the crystal lattice through the long-range Coulomb force, causing a local distortion of the surrounding ionic lattice. The combined entity consisting of the electron (or hole) and its self-induced lattice distortion behaves as a quasiparticle known as the Fröhlich polaron. This description is particularly important for ionic and highly polar crystals, such as II–VI semiconductors, alkali halides, metal oxides, and other polar materials, where the long-range interaction between conduction electrons and longitudinal optical (LO) phonons gives rise to strong electron–phonon coupling, commonly referred to as the Fröhlich interaction.

The strength of this interaction is characterized by the dimensionless Fröhlich coupling constant, α, whose magnitude determines the extent of the electron–phonon interaction. Depending on the material, α may vary from weak to strong coupling regimes. Experimentally measured values of α range from approximately 0.02–0.2 for common III–V semiconductors, 0.3–0.5 for II–VI semiconductors, typically 2–4 for alkali halides, and can be as high as 4.5 in perovskite materials such as $SrTiO_3$ [12].

The formation of a Fröhlich polaron increases the effective mass of the charge carrier relative to that of the bare electron, and the degree of this mass enhancement depends directly on the coupling constant α. Physically, the parameter α is approximately proportional to twice the average number of LO phonons comprising the phonon cloud surrounding a given electron or hole [13]. Consequently, polaronic effects become increasingly significant when the Fröhlich coupling constant approaches or exceeds unity, leading to pronounced modifications in the transport, optical, and nonlinear properties of polar semiconductor materials. These effects play an important role in determining the performance of semiconductor-based nonlinear optical and optoelectronic devices.

Compound semiconductors constitute a broad and technologically important class of materials owing to their exceptional electrical, optical, and transport properties. Among these, group III–V semiconductors, such as GaAs, and group II–VI semiconductors, such as ZnS, have attracted considerable attention because of their direct band-gap nature and sufficiently strong polar character, which gives rise to significant electron–phonon interactions. These characteristics make them highly suitable for a wide range of optoelectronic and nonlinear photonic applications, including lasers, light-emitting diodes, optical modulators, photodetectors, and parametric devices.

In the present study, GaAs and ZnS have been selected as representative polar compound semiconductors to investigate the influence of the Fröhlich coupling constant on the operational characteristics of optical parametric interactions. A comparative analysis has been carried out to examine how variations in the electron–phonon coupling strength affect the threshold pump field and the efficiency of the parametric process. The study provides valuable insights into the role of the Fröhlich interaction in determining the nonlinear optical response of polar semiconductor media and offers useful guidelines for selecting suitable materials for high-performance optoelectronic and photonic devices.

2. Theoretical formulations

This section presents the theoretical formulation for evaluating the threshold pump intensity required to initiate Fröhlich interaction-based optical parametric interaction in a highly doped polar semiconductor medium. The analysis is developed using the hydrodynamic model of semiconductor plasma together with the coupled-mode approach, taking into account the nonlinear interaction between free carriers and longitudinal optical phonons mediated through the Fröhlich electron–phonon coupling. The threshold condition is obtained by examining the onset of parametric amplification under the appropriate phase-matching conditions.

For the theoretical analysis, a spatially uniform pump electric field ($|\vec{k}_0| \approx 0$) is assumed to propagate through the semiconductor medium and is represented as:

$$\vec{E}_0 = \hat{x}\exp(-i\omega_0 t) .$$

The pump field is applied to a homogeneous, highly doped semiconductor medium subjected to a uniform external magnetostatic field $\vec{B}$ directed along the *y*-axis. The pump wave is assumed to be spatially uniform, which is justified when the wavelengths of the excited waves are much smaller than the characteristic scale length over which the pump field varies (i.e. $k_0 << k$, so that k_0 may be neglected with respect to k). Under this condition, the spatial variation of the pump field may be neglected in comparison with that of the excited waves, thereby considerably simplifying the theoretical analysis.

The optical parametric interaction is assumed to satisfy the phase-matching conditions for both momentum and energy conservation (i.e. $k_0 = k_1 \pm k_s$ and $\omega_0 = \omega_1 \pm \omega_s$), ensuring efficient nonlinear coupling among the pump wave, polaron mode, and signal wave. These conditions govern the coherent transfer of energy from the pump wave to the generated waves and are essential for the onset of parametric amplification.

To investigate the threshold characteristics of the polaron mode in magnetized semiconductor media, the analysis is carried out within the framework of the hydrodynamic model of a homogeneous semiconductor plasma of infinite extent. The nonlinear interaction is described through the effective third-order optical susceptibility χ_{eff}, which originates from the Fröhlich electron–phonon coupling and the nonlinear motion of the free carriers. The hydrodynamic description replaces the microscopic carrier dynamics by macroscopic variables such as carrier density and average velocity while retaining the essential physics of the nonlinear interaction.

The validity of the present formulation is restricted to the hydrodynamic regime, where the product of the wave number *k* and the carrier mean free path *l* satisfies the condition: $kl << 1$. Within this limit, carrier transport can be adequately described by the fluid approximation, making the model well suited for analyzing Fröhlich interaction-based optical parametric processes in highly doped magnetized semiconductor media.

To obtain a simplified expression for the second-order optical susceptibility, the coupled-mode approach is employed by evaluating the nonlinear polarization induced in the semiconductor medium. The nonlinear interaction between the pump, signal, and polaron waves gives rise to an induced depolarizing field, which strongly couples the longitudinal and transverse dynamical modes of the medium. As a consequence of this coupling, the natural resonance of the charge carriers is shifted from the pure electron-cyclotron frequency, leading to the excitation of a collective cyclotron mode.

The corresponding resonant frequency $\omega_{cc} = (\omega_0^2 + \omega_c^2)^{1/2}$ is determined by the combined influence of the electron-cyclotron frequency $\omega_c = (-eB/m_e)$, the electron-plasma frequency $\omega_p = (n_0 e^2 / m_e \varepsilon)^{1/2}$, and the high-frequency dielectric constant $\varepsilon(=\varepsilon_0 \varepsilon_s)$ of the semiconductor. This collective resonance plays a central role in enhancing the nonlinear interaction responsible for the optical parametric process. Here, the electron-cyclotron frequency is determined by the applied magnetic field, the electron-plasma frequency depends on the equilibrium carrier concentration, the high-frequency dielectric constant characterizes the dielectric response of the medium, and m_e denotes the effective electron mass.

The theoretical formulation is based on the following fundamental equations governing carrier transport, charge continuity, lattice dynamics, and electromagnetic wave propagation in the semiconductor medium:

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0 \tag{1}$$

$$\frac{\partial E_{pl}}{\partial x} = -\frac{n_1 e}{\varepsilon_0} + \left(\frac{Nq}{\varepsilon_0}\right)\frac{\partial R}{\partial x} \tag{2}$$

The continuity equation, given by Eq. (1), expresses the conservation of electric charge in the semiconductor plasma. Here, n_0 and n_1 denote the equilibrium and perturbed electron densities, respectively, while v_0 and v_1 represent the corresponding oscillatory electron fluid velocities. The electron dynamics are described in terms of a fluid consisting of carriers with an effective mass m_e.

The electrostatic field, E_{pl}, arises from the combined effects of the induced electronic polarization and the lattice polarization associated with the Fröhlich electron–phonon interaction. This field is obtained from Poisson's equation, given by Eq. (2), where ε_0 denotes the permittivity of free space. The induced electrostatic field plays a crucial role in coupling the carrier-density fluctuations with the lattice vibrations, thereby facilitating the nonlinear interaction responsible for the optical parametric process.

$$\frac{\partial^2 \vec{r}}{\partial t^2} + (\omega_p^2 + \omega_c^2)\vec{r} + 2\Gamma_e \frac{\partial \vec{r}}{\partial t} = -\frac{e}{m_e}\left(\vec{E}_0 + \frac{\partial \vec{r}}{\partial t} \times B\right) \tag{3}$$

$$\frac{\partial^2 R}{\partial t^2} + (\omega_p^2 + \omega_c^2)R + 2\Gamma_{pl} \frac{\partial R}{\partial t} = \frac{q}{M_{pl}} E_{pl} \, . \tag{4}$$

In the above formulation, Γ_e represents the electron collision frequency, which accounts for momentum relaxation arising from carrier scattering processes. The parameter $\Gamma_{pl} = \Gamma_e + \Gamma_{ph}$ denotes the effective damping constant associated with the polaron mode and incorporates the decay of longitudinal optical (LO) phonons. It is commonly expressed in terms of the optical phonon decay constant $\Gamma_{ph} = 10^{-2}\omega_{pl}$, thereby accounting for the finite lifetime of lattice vibrations.

The quantities E_0 and *B* represent the pump electric field and the externally applied magnetostatic field, respectively. The electrostatic field E_{pl} denotes the effective polaron field generated by the coupled oscillations of free carriers and the polar lattice. This field mediates the nonlinear interaction between the electromagnetic wave and the lattice vibrations, and therefore plays a fundamental role in the Fröhlich interaction-based optical parametric process.

The parameter *q* represents the effective ionic charge associated with the polar lattice vibrations and is given by:

$$q = \omega_L \left[\frac{M}{N}\varepsilon_0\left(\frac{1}{\varepsilon} - \frac{1}{\varepsilon_s}\right)\right]^{1/2} . \tag{5}$$

Here, *M* denotes the reduced mass of the diatomic basis of the crystal lattice, while $N(=a^3)$ represents the number of unit cells per unit volume. The parameter *a* is the lattice constant of the semiconductor crystal. These quantities determine the effective ionic charge associated with the lattice vibrations and hence govern the strength of the electron–phonon interaction.

The quantity M_{pl} represents the effective mass of the Fröhlich polaron, which is slightly greater than the effective mass of a quasi-free electron. This increase in effective mass arises because a moving electron is accompanied by the polarization cloud generated through its interaction with the surrounding lattice. Consequently, the electron must carry this lattice distortion as it propagates through the crystal, resulting in an increase in its inertia and, therefore, its effective mass.

For weak- and intermediate-coupling regimes, an approximate analytical expression for the polaron effective mass was derived by Richard Feynman using his variational path-integral approach [14]. This expression relates the effective polaron mass to the Fröhlich coupling constant and provides an accurate estimate of the mass enhancement caused by electron–phonon interactions. It is employed in the present analysis to account for the influence of the Fröhlich interaction on the nonlinear optical response of the semiconductor medium.

$$M_{pl} \approx m_e\left(1 + \frac{\alpha}{6}\right) \qquad (\text{For } \alpha << 1)$$

The Fröhlich coupling constant is a dimensionless parameter that quantifies the strength of the long-range interaction between a charge carrier and longitudinal optical (LO) phonons in a polar semiconductor. It plays a fundamental role in determining the degree of electron–phonon coupling, the formation of polarons, and the resulting transport and nonlinear optical properties of the material.

A larger value of the Fröhlich coupling constant corresponds to stronger electron–phonon interaction, leading to a more pronounced lattice polarization around the moving charge carrier and consequently a greater enhancement of the polaron effective mass. Conversely, materials with smaller values of Fröhlich coupling constant exhibit weaker carrier–phonon coupling, allowing charge carriers to retain higher mobility and lower effective inertia. Therefore, the magnitude of the Fröhlich coupling constant significantly influences the efficiency of nonlinear optical processes and the threshold conditions for optical parametric interactions in polar semiconductor media.

The Fröhlich coupling constant is given by:

$$\alpha = \frac{1}{2}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\right)\frac{e^2}{\hbar\omega}\left(\frac{2m\omega_{LO}}{\hbar}\right)^{1/2} . \tag{7}$$

The magnitude of the Fröhlich coupling constant is a material-dependent parameter and therefore varies significantly from one polar semiconductor to another. Depending on the crystal structure and dielectric properties, the electron–phonon interaction may belong to either the weak-coupling or strong-coupling regime. In the present work, the analysis is confined to the weak electron–phonon coupling regime ($\alpha << 1$), where the Fröhlich polaron retains a quasi-free-particle character and the hydrodynamic description remains valid. Under this condition, the perturbative treatment of the electron–phonon interaction provides an accurate description of the nonlinear optical response of the semiconductor medium.

Following the theoretical procedure adopted by Neogi and Ghosh [15], and employing Eqs. (1), (3), and (4), the governing equation describing the coupled carrier–lattice dynamics can be derived as follows:

$$\frac{\partial^2 n_1}{\partial t^2} + 2\Gamma_e \frac{\partial n_1}{\partial t} + \bar{\Omega}_p^2 n_1 A_1 Z - \omega_{p,pl}^2 n_0 A_1 \frac{\partial R}{\partial x} - 2\Gamma_{ph} T = -ikn_1 A_2 \bar{E} \tag{8}$$

where

$$\bar{E} = \left(-\frac{e}{m}\right)\frac{\omega_0^2}{\omega_0^2 - \omega_c^2} E_0 , \quad A_1 = \frac{\omega_{0,pl}^2}{\omega_{0p}^2 - \omega_p^2 - \omega_c^2} ,$$

$$A_2 = \frac{\omega_0^2}{\omega_0^2 - \Omega_p^2 - \omega_c^2} , \quad \omega_{p,pl}^2 = \frac{Nq^2}{M\varepsilon_0} ,$$

$$Z = \frac{qm}{eM_{pl}} , \quad \bar{\Omega}_p^2 = Z\omega_p^2 ,$$

$$T = n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_0}{\partial x} .$$

Here, $\omega_{0,pl}$ denote the frequencies of the coupled normal modes in the presence of an externally applied magnetic field. These hybrid modes arise from the coupling between the collective cyclotron excitations of the free carriers and the longitudinal optical (LO) phonons through the macroscopic longitudinal electric field generated within the polar semiconductor. Such coupled excitations represent the magneto-polaron modes of the system and play a crucial role in determining its nonlinear optical response.

The frequencies of these coupled modes depend on both the electron-cyclotron frequency and the electron-plasma frequency, and are commonly designated as the upper (ω_+) and lower (ω_-) magneto-polaron branches. Their expressions are obtained by solving the coupled carrier–phonon equations in the presence of the magnetic field and are given by [16]:

$$\omega_{0,pl} = \left[\frac{\Omega_p^2 + \omega_c^2 + \omega_L^2 + \left[(\Omega_p^2 + \omega_c^2 + \omega_L^2)^2 - 4(\Omega_p^2\omega_T^2 + \omega_c^2\omega_L^2)\right]^{1/2}}{2}\right]^{1/2} . \tag{9}$$

To simplify the coupled dynamical equations, the Rotating Wave Approximation (RWA) is employed. Under this approximation, the rapidly oscillating (non-resonant) terms, whose time averages vanish over an optical cycle, are neglected, while only the slowly varying resonant terms are retained. Since these slowly varying components are primarily responsible for the energy exchange among the interacting waves, the RWA considerably simplifies the analysis without affecting the essential physics of the optical parametric interaction.

Applying the Rotating Wave Approximation to Eq. (8) and performing the necessary algebraic simplifications, the expression for the slowly varying component of the carrier-density perturbation is obtained as follows:

$$n_s = ikn_0\left(\frac{q}{M_{pl}}\right)QG\bar{E}_{pl}\omega_{p,pl}^2 . \tag{10}$$

In the present work, the contribution of the transition dipole moment has been neglected in order to isolate and examine the role of the nonlinear current density in the generation of the induced polarization of the semiconductor medium. This assumption is justified because the pump photon energy is considered to be well below the fundamental band-gap energy, thereby suppressing interband transitions and minimizing the influence of dipole-transition mechanisms. Consequently, the nonlinear optical response of the medium is assumed to arise predominantly from the nonlinear motion of free carriers mediated through the Fröhlich electron–phonon interaction.

Under these conditions, the induced nonlinear current density associated with the optical parametric interaction can be expressed as:

$$J(\omega_1) = -ev_0 n_s^* . \tag{11}$$

Here, the superscript *, denotes the complex conjugate of the corresponding quantity. Substituting the expressions obtained for the carrier-density perturbation and the oscillatory electron velocity into the above equation, and carrying out the necessary algebraic simplifications, the induced nonlinear current density can be written in the following form:

$$J(\omega_1) = \frac{qk\varepsilon_0}{M_{pl}}\left(\frac{\omega_0\omega_p^2\omega_{p,pl}^2}{\omega_0^2-\omega_c^2}\right)Q^*G^*E_0E_{pl}^* , \tag{12}$$

where,

$$Q = \frac{A_1}{-\omega_{0,pl}^2 - \omega_c^2 - 2i\omega_{0,pl}\Gamma_{pl}} ,$$

$$G = \left[(\delta_2^2 - 2i\omega_{0,pl}\Gamma_e) - \frac{A_2^2k^2\left|\bar{E}\right|^2}{\delta_1^2 + 2i\Gamma_e\omega_1}\right]^{-1} ,$$

$$\delta_1^2 = A_1\bar{\Omega}_p^2 - \omega_1^2 ,$$

$$\delta_2^2 = A_1\bar{\Omega}_p^2 - \omega_{0,pl}^2 .$$

The coupling between the collective cyclotron excitations of the free carriers and the longitudinal optical (LO) phonons gives rise to an induced polarization within the semiconductor medium. This polarization is local in nature and originates from the displacement of ions from their equilibrium lattice positions under the influence of the interacting electromagnetic fields. The induced lattice polarization, together with the electronic polarization, constitutes the fundamental mechanism responsible for the nonlinear optical response associated with the Fröhlich interaction.

Treating the nonlinear polarization, P_{nl}, as the time integral of the induced nonlinear current density, $J(\omega_1)$, and applying the Rotating Wave Approximation (RWA) to retain only the slowly varying resonant terms while neglecting the rapidly oscillating non-resonant components, the induced nonlinear polarization of the medium can be expressed as:

$$P_{nl} = \int J(\omega_1)\,dt \tag{13}$$

Substituting the expression for the induced nonlinear current density obtained from Eq. (9) into the above relation for the nonlinear polarization, and performing the necessary algebraic simplifications under the Rotating Wave Approximation (RWA), the induced nonlinear polarization of the semiconductor medium is obtained as:

$$P_{nl} = \frac{-iqk\varepsilon_0}{M_{pl}\omega_1}\left(\frac{\omega_0\omega_p^2\omega_{p,pl}^2}{\omega_0^2-\omega_c^2}\right)Q^*G^*E_0E_{pl}^* . \tag{14}$$

To determine the threshold pump amplitude required for the onset of the optical parametric process, the condition corresponding to the transition from absorption to amplification is imposed by setting the net gain equal to zero, i.e., $P_{nl} = 0$. This condition represents the minimum pump field at which the nonlinear interaction exactly compensates the intrinsic losses of the medium, thereby initiating parametric amplification.

Substituting this threshold condition into the governing equation for the growth rate and carrying out the necessary mathematical simplifications, the expression for the threshold pump amplitude is obtained as:

$$E_{0th} = \left|\frac{m_e(\omega_0^2-\omega_c^2)}{ekA_2\omega_0^2}\delta_1\delta_2\right| . \tag{15}$$

3. Result and discussion

The theoretical formulation developed in the preceding section has been employed to perform a detailed numerical investigation of the threshold characteristics of the proposed Fröhlich interaction-based optical parametric process in two representative polar compound semiconductors. The primary objective of this analysis is to compare the performance of these materials and identify the conditions under which a minimum threshold pump field can be achieved through the appropriate selection of external control parameters, such as the applied magnetic field**,** carrier concentration, and wave vector.

For this purpose, ZnS (Group II–VI) and GaAs (Group III–V) have been selected as representative semiconductor materials because of their technological importance and distinct electron–phonon coupling strengths. The material parameters employed in the numerical calculations are summarized in Table 1. Both semiconductor crystals are assumed to be irradiated by a 10.6 μm pulsed CO_2 laser, whose photon energy lies well below the band-gap energies of the materials. Under this condition, the optical response is dominated by free-carrier and lattice interactions rather than interband transitions, thereby enabling the required nonlinear interaction through the Fröhlich electron–phonon coupling. The numerical analysis presented in this section provides a comparative assessment of the threshold behavior of ZnS and GaAs and highlights the influence of the Fröhlich coupling constant on the efficiency of the optical parametric interaction.

Table 1: Material parameters of the polar compound semiconductors ZnS and GaAs used for the numerical analysis of the Fröhlich interaction-based optical parametric process.

Crystal	m_e/m_0	α	$\omega_{LO}(s^{-1})$	ε_s	ε_{opt}
GaAs (III-V)	0.066	0.068	5.548×10^{13}	12.9	10.9
ZnS (II-VI)	0.28	0.63	6.837×10^{13}	5.1	2.95

Equation (15) describes the dependence of the threshold pump field on the externally applied magnetic field, carrier concentration, and wave vector of the semiconductor medium. Using the material parameters listed in Table 1, numerical calculations have been carried out to investigate the influence of these external parameters on the threshold electric field, E_{0th}, required to initiate the Fröhlich interaction-based optical parametric process. The corresponding results are presented in Figs. 1–3.

In all the figures, the dotted curve represents the results for ZnS, whereas the solid curve corresponds to GaAs.

Figure 1 illustrates the variation of the threshold pump electric field, E_{0th}, as a function of the wave vector k of the polaron mode. A similar trend is observed for both semiconductor crystals, namely that the threshold field decreases with increasing wave vector. This behavior indicates that larger wave vectors enhance the nonlinear coupling between the interacting waves, thereby facilitating the onset of the parametric process at lower pump intensities. However, a significant quantitative difference exists between the two materials. The threshold pump field required for ZnS is found to be approximately one order of magnitude higher than that required for GaAs over the entire range of wave vectors considered. This difference may be attributed to the comparatively stronger electron–phonon coupling and larger Fröhlich coupling constant in ZnS, which increases the effective polaron mass and consequently requires a higher pump field to initiate the nonlinear interaction. These results suggest that GaAs, owing to its lower threshold requirement, is a more suitable material for realizing efficient and energy-effective optical parametric interactions.

Figure 2 illustrates the variation of the threshold pump electric field, E_{0th}, with the externally applied magnetic field for both ZnS and GaAs. It is evident that the threshold field decreases with increasing magnetic field strength in both semiconductor crystals. The applied magnetic field modifies the carrier dynamics through the electron-cyclotron motion, thereby strengthening the nonlinear coupling responsible for the optical parametric interaction and reducing the pump field required to initiate the process.

Among the two materials investigated, GaAs exhibits the lowest threshold value, reaching a minimum of approximately 7.422×10^5 Vm^{-1}, whereas the corresponding minimum threshold field for ZnS is about 1.661×10^6 Vm^{-1}. These results clearly indicate that the parametric interaction can be initiated more efficiently in GaAs than in ZnS under similar operating conditions.

The comparatively higher threshold field observed for ZnS may be attributed to its larger Fröhlich coupling constant, which enhances the electron–phonon interaction and increases the effective polaron mass. As a result, a stronger pump field is required to overcome the increased inertia of the polaron and initiate the nonlinear optical process. Conversely, the relatively weaker electron–phonon coupling in GaAs leads to a lower effective polaron mass and, consequently, a significantly reduced threshold pump field. Therefore, the present analysis demonstrates that materials with lower Fröhlich coupling constants are more suitable for achieving low-threshold and energy-efficient optical parametric interactions, making GaAs a more promising candidate than ZnS for practical nonlinear optoelectronic applications.

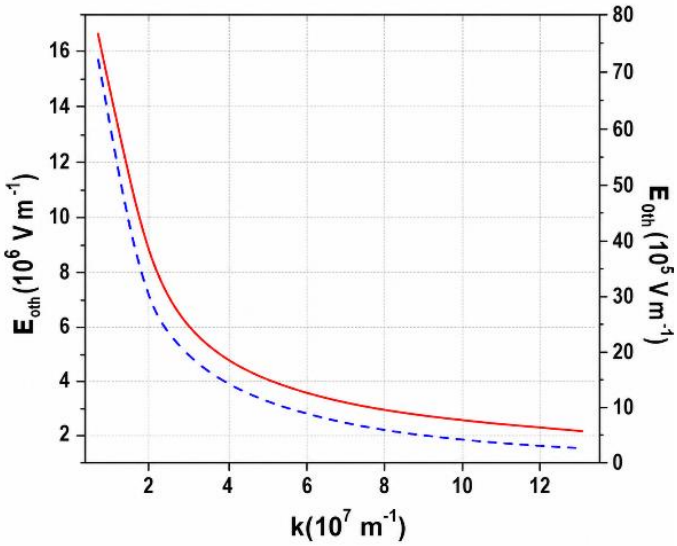

Figure 1: Variation of threshold electric field E_{0th} with number k at B =13 T and $n_0 = 4.1\times10^{24}$ m^3.

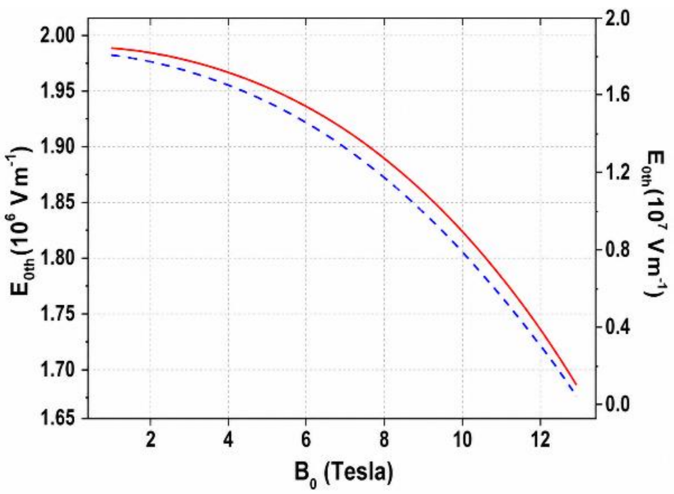

Figure 2: Variation of threshold electric field E_{0th} with magnetic field B at $k = 10^8 m^{-1}$ and $n_0 = 4.1\times10^{24}$ m^3.

Figure 3 shows the variation of the threshold pump electric field, E_{0th}, as a function of the equilibrium carrier concentration n_0 for GaAs and ZnS. Both materials exhibit qualitatively similar behavior. At low doping concentrations, the threshold pump field is relatively high because the free-carrier density is insufficient to support efficient nonlinear coupling between the electromagnetic wave and the polaron mode. As the carrier concentration increases, the electron-plasma frequency increases, thereby strengthening the nonlinear interaction and reducing the pump field required to initiate the optical parametric process.

For GaAs, the threshold pump field decreases continuously with increasing carrier concentration and attains a minimum value of approximately 1.428×10^5 Vm^{-1} at an equilibrium carrier concentration of 4.1×10^{24} m^{-3}. Similarly, for ZnS, the threshold field reaches its minimum value of about 3.957×10^{6} Vm^{-1} at a carrier concentration of 5.0×10^{24} m^{-3}.

A comparison of the two materials clearly indicates that GaAs requires a substantially lower threshold pump field than ZnS over the entire range of carrier concentrations investigated. This behavior is consistent with the earlier observations and may be attributed to the comparatively smaller Fröhlich coupling constant of GaAs, which results in a lower polaron effective mass and facilitates the onset of the nonlinear parametric interaction. In contrast, the stronger electron–phonon coupling in ZnS increases the effective inertia of the polaron, thereby requiring a higher pump field to achieve parametric excitation. These results further confirm that materials with weaker Fröhlich electron–phonon coupling are more favorable for realizing low-threshold, energy-efficient optical parametric devices.

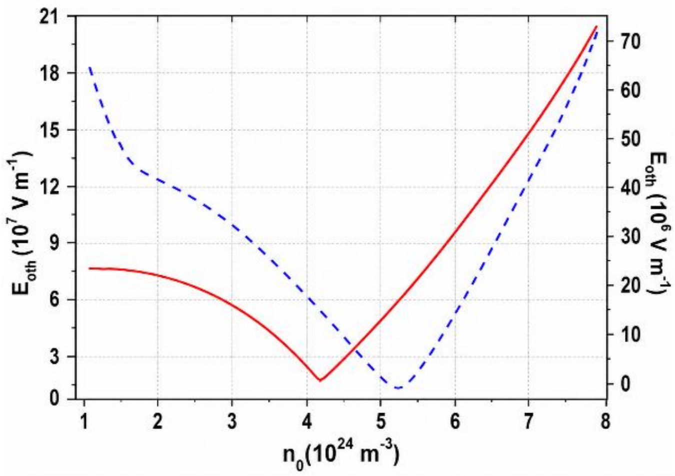

Figure 3: Variation of threshold electric field E_{0th} with carrier concentration at $k = 10^{8}$ m^{-1} and B = 13 T.

The observed variation may be attributed to the resonance condition $(\omega_p^2 \approx \omega_1^2 - \omega_c^2)$, under which the electron-plasma frequency approaches the characteristic frequency of the nonlinear interaction. As the carrier concentration increases, the electron-plasma frequency also increases, thereby strengthening the coupling between the electromagnetic wave and the polaron mode. Consequently, the threshold pump field decreases and reaches its minimum value when the resonance condition is satisfied. At this point, the nonlinear interaction becomes most efficient, and the minimum pump field is required to initiate the optical parametric process.

A further increase in the carrier concentration shifts the system away from the resonance condition, resulting in an increase in the threshold pump field. In this regime, the enhanced plasma response reduces the efficiency of the nonlinear coupling, and a stronger pump field is required to sustain the parametric interaction.

The results also reveal that the Fröhlich coupling constant has a pronounced influence on the threshold characteristics. Materials possessing a larger coupling constant exhibit the threshold minimum at higher carrier concentrations, and the corresponding minimum threshold field is also significantly larger. This behavior is a consequence of the stronger electron–phonon interaction, which increases the effective polaron mass and consequently demands greater pump energy to excite the nonlinear process.

Among the two semiconductor materials investigated, GaAs consistently exhibits a substantially lower threshold pump field than ZnS over the entire range of carrier concentrations. The comparatively smaller Fröhlich coupling constant of GaAs results in weaker electron–phonon coupling, a lower effective polaron mass, and therefore a more efficient nonlinear interaction. These observations further establish GaAs as the more favorable material for realizing low-threshold and energy-efficient optical parametric devices.

4. Conclusions

A comprehensive theoretical framework has been developed based on the hydrodynamic model of semiconductor plasma and the coupled-mode theory to investigate Fröhlich interaction-based optical parametric interaction in polar compound semiconductors. The analytical formulation enabled the determination of the threshold pump field required to initiate the parametric process and to examine the influence of important material and external parameters on its operational characteristics.

To validate the proposed model, detailed numerical estimations were carried out for two technologically important compound semiconductors, GaAs and ZnS**,** irradiated by a 10.6 μm CO_2 laser. The analysis demonstrates that the application of a sufficiently strong external magnetic field considerably reduces the threshold pump field by enhancing the nonlinear coupling between the free carriers and longitudinal optical phonons. Likewise, heavily doped semiconductor crystals are found to be more suitable for achieving efficient parametric interaction because the increased carrier concentration strengthens the nonlinear response of the medium. However, the carrier concentration must remain below the critical value for which the electron-plasma frequency exceeds the pump-wave frequency; otherwise, the incident pump wave is reflected by the medium, thereby suppressing the nonlinear interaction.

A comparative analysis of GaAs and ZnS reveals the significant role played by the Fröhlich coupling constant in determining the threshold characteristics. Owing to its Fröhlich coupling constant being approximately an order of magnitude larger than that of GaAs, ZnS requires a threshold pump field that is nearly ten times higher than that required for GaAs. The stronger electron–phonon coupling in ZnS increases the effective polaron mass and consequently demands greater pump energy to initiate the optical parametric process.

The present study therefore establishes that the threshold characteristics are strongly governed by the Fröhlich coupling constant. Materials possessing smaller values of the Fröhlich coupling constant, such as GaAs, exhibit significantly lower threshold pump fields and are consequently more attractive for the realization of low-power, energy-efficient, and cost-effective optical parametric devices. These findings provide useful guidelines for the selection and optimization of semiconductor materials for future nonlinear optoelectronic and photonic applications.

Authors' contributions

The authors read and approved the final manuscript.

Conflicts of interest

The authors declares no conflict of interest.

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Data availability

No new data were created.

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