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Original Research Article

Modulational amplification via nonlinear plasmon–transverse optical phonon interaction in weakly polar magnetoactive doped III–V semiconductors

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ABSTRACT
Using the hydrodynamic model of semiconductor plasma and the coupled-mode theoretical approach, a detailed analytical investigation of plasmon–transverse optical phonon (PL–TOP) interaction-induced modulational amplification has been carried out in weakly polar, magnetoactive, doped III–V semiconductor crystals. The nonlinear interaction mechanism has been attributed to the effective third-order optical susceptibility arising from the nonlinear induced current density of the semiconductor medium. Analytical expressions for the threshold pump amplitude required for the onset of modulational amplification and the corresponding growth rate of the modulated wave have been derived. Numerical calculations have been performed for an n-type InSb crystal at 77 K, irradiated by a pulsed 10.6 μm CO_2 laser. The influence of important system parameters, including the wave number, the externally applied magnetic field (through the electron-cyclotron frequency), and the doping concentration (through the electron-plasma frequency), on the threshold condition and amplification characteristics has been systematically analyzed. The results reveal that an appropriate choice of the externally applied magnetic field and carrier concentration can significantly reduce the threshold pump amplitude while enhancing the growth rate of the modulated wave. The present analysis highlights the potential of weakly polar, magnetoactive, heavily doped III–V semiconductor crystals as promising nonlinear optical media for realizing modulational instability-based amplification and advanced optoelectronic applications.

1. Introduction

Nonlinear optics is an advanced field of optical science that focuses on the interaction of intense coherent electromagnetic radiation with matter, resulting in the generation of new electromagnetic waves with modified characteristics such as amplitude, frequency, phase, and polarization [1,2]. Since the advent of high-intensity laser sources, nonlinear optical phenomena have attracted considerable research interest due to their significant applications in laser technology, optical signal processing, and high-speed optical communication systems [3].

For the development and optimization of modern optoelectronic devices, particularly optical modulators, photonic switches, and communication components, a comprehensive understanding of nonlinear interactions occurring within optical media is essential. Among various nonlinear phenomena, modulational interactions have gained considerable importance because of their ability to control and amplify optical signals, thereby offering promising applications in advanced and ultrafast optical communication technologies [4–7].

Modulational interaction generally refers to the instability of a wave propagating through a nonlinear dispersive medium, where an initially stable continuous-wave state becomes unstable due to nonlinear effects and evolves into a temporally modulated structure [8]. This phenomenon, commonly known as modulational instability (MI) or sideband instability, occurs when small perturbations introduced into a periodic waveform are amplified through nonlinear coupling. As a consequence, new spectral sidebands are generated, and the original continuous waveform can eventually transform into a sequence of localized pulses [9–11].

In its simplest representation, modulational instability arises from the nonlinear interaction between a strong carrier wave of frequency ω_0 and weak perturbation waves (sidebands) with frequencies $\omega_0 + \omega$ and $\omega_0 - \omega$. This process can be considered as a special case of four-wave mixing, where two photons of the pump wave interact through the nonlinear medium to generate two new photons corresponding to the upper and lower sideband frequencies.

The phenomenon of modulational instability and related nonlinear wave interactions has been extensively investigated theoretically by several researchers because of its significant scientific and technological importance [12–14]. Among various nonlinear optical systems, the study of wave amplification, attenuation, and frequency mixing in semiconductor media, particularly in III–V compound semiconductors, has gained considerable attention due to its direct relevance to the development of advanced optical communication and photonic devices.

The unique importance of semiconductor crystals arises from the availability of a large population of free charge carriers and the possibility of controlling carrier dynamics through optical excitation and external perturbations. In III–V semiconductor materials, parameters such as doping concentration, material composition, and microstructural engineering have been extensively utilized to optimize the performance and efficiency of optoelectronic devices [15]. Furthermore, the nonlinear optical response of these semiconductor systems can be effectively controlled and modified by applying external electric and magnetic fields [16–18].

The dependence of nonlinear optical properties on externally applied fields provides an effective mechanism for understanding and controlling the physical processes involved in modulational interactions. Considering the significant role of III–V semiconductor crystals in modern optoelectronic technology, detailed analytical investigations of modulational phenomena in these materials are of considerable importance. Such studies become particularly relevant in semiconductor systems supporting multiple modes of optical excitation, where energy exchange among interacting modes may result in either amplification or attenuation of the generated waves.

III–V semiconductor crystals support a variety of elementary excitations, including plasmons (PLs), acoustical phonons (APs), and optical phonons (OPs), which give rise to several types of coupled plasmon–phonon interaction processes. These interactions play a crucial role in determining the optical response, carrier dynamics, and transport properties of electrons in semiconductor materials.

The interaction of low-frequency acoustical phonons with electrons occurs through the strain field generated by lattice vibrations and is generally described in terms of the deformation potential interaction. Similarly, optical phonons can be considered as sources of internal lattice strain, and their coupling with charge carriers can also be represented through the optical phonon deformation potential. In polar semiconductor materials, both low-frequency acoustical phonons and optical phonons can generate electric fields due to the displacement of charged ions associated with lattice vibrations, thereby strongly influencing the nonlinear optical response of the medium.

When an intense electromagnetic pump wave propagates through a semiconductor crystal, it induces a strong electrostatic space-charge field oscillating at the optical phonon frequency. This induced field contains both longitudinal and transverse components. In weakly polar III–V semiconductor crystals, the longitudinal optical phonons (LOPs) are associated predominantly with longitudinal electric fields, whereas transverse optical phonons (TOPs) are coupled with transverse electric fields. For finite wave numbers, the excitation strengths of LOPs and TOPs are governed by two independent mechanisms, namely the electro-optic interaction and the deformation potential Raman tensor, respectively [19].

In III–V semiconductor crystals with zinc-blende structures, such as InSb, GaAs, GaSb, and InAs, the polarization contribution associated with LOP modes is generally stronger than that of TOP modes [20]. Consequently, most previous theoretical investigations of modulational interactions in weakly polar III–V semiconductors have primarily focused on the contribution of longitudinal optical phonons while neglecting the influence of transverse optical phonon fields [21–28].

However, for a complete understanding of nonlinear wave coupling and modulational amplification mechanisms in weakly polar III–V semiconductor systems, it is essential to consider the deformation potential interaction between plasmons and transverse optical phonons. Inclusion of this interaction provides a more comprehensive description of the nonlinear response and may reveal additional possibilities for controlling modulational processes in semiconductor-based optoelectronic devices.

Considering the significant role of modulational interactions and various plasmon–phonon coupling mechanisms in III–V semiconductor crystals, the present work focuses on the investigation of plasmon–transverse optical phonon (PL–TOP) interaction-induced modulational amplification in weakly polar, magnetized, doped III–V semiconductor materials. The analysis has been carried out using the hydrodynamic model of one-component semiconductor plasma in conjunction with the coupled-mode theoretical approach.

The nonlinear interaction mechanism originates from the deformation potential coupling between free carriers, represented by the electron-plasma wave (plasmon), and the transverse optical phonon modes associated with molecular lattice vibrations. An intense pump field generates a space-charge electric field oscillating at the TOP frequency, which interacts with the modulated signal wave in the presence of an externally applied magnetic field. Under suitable phase-matching conditions, this nonlinear coupling facilitates energy transfer from the pump wave to the modulated mode, resulting in its amplification.

The strength of the modulational interaction is characterized by the effective third-order optical susceptibility, $\chi^{(3)}$, of the semiconductor medium, which represents the nonlinear optical response arising from the induced current density associated with the PL–TOP coupling process.

The present analytical investigation has been developed under the following assumptions to simplify the theoretical formulation while retaining the essential physical characteristics of the nonlinear interaction process:

1. All interacting waves are considered as infinite plane waves.
2. The interacting modes satisfy the required phase-matching conditions for efficient energy transfer.
3. The depletion of energy from the incident pump wave during the interaction process is neglected, assuming that the pump field remains approximately constant.
4. The semiconductor medium is assumed to be homogeneous and isotropic, possessing a non-degenerate parabolic energy band structure at a moderately low temperature of 77 K.
5. The photon energy of the pump radiation is considered to be significantly lower than the band-gap energy of the semiconductor crystal. This assumption ensures that the optical response of the medium is governed mainly by free-carrier effects while avoiding significant contributions from photo-induced interband electronic transitions.

To validate the proposed theoretical model, an extensive numerical analysis has been carried out using realistic material parameters corresponding to a weakly polar III–V semiconductor crystal (n-type InSb) irradiated by a frequency-doubled, pulsed 10.6 μm CO_2 laser. The obtained numerical results demonstrate the influence of relevant physical parameters on the PL–TOP interaction-based modulational amplification process.

The organization of the paper is as follows:

In Section 2, the theoretical formulation of the PL–TOP interaction-induced modulational amplification is developed.

Analytical expressions for the effective third-order optical susceptibility $\chi_e^{(3)}$, the threshold pump amplitude $(E_{0,th})_{pl-top}$ required for initiating modulational amplification, and the growth rate $(g)_{pl-top}$ of the modulated wave are derived by employing the single-component hydrodynamic plasma model along with the coupled-mode theory. Section 3 presents the numerical evaluation of the threshold pump amplitude and growth rate by considering the influence of important controlling parameters, particularly the externally applied magnetic field and the doping concentration of the semiconductor crystal. The numerical analysis is performed using appropriate material parameters for the selected III–V semiconductor medium. The major findings and significant conclusions obtained from the present analytical investigation are summarized in Section 4.

2. Theoretical formulations

For the theoretical investigation of modulational interaction and the subsequent amplification of modulated waves in weakly polar, magnetized, doped III–V semiconductor crystals, the well-established hydrodynamic model of homogeneous n-type semiconductor plasma has been employed. The semiconductor medium is considered at 77 K (liquid nitrogen temperature), with electrons acting as the charge carriers under the influence of an intense pump wave and an externally applied magnetostatic field while maintaining thermal equilibrium conditions.

The proposed formulation describes the nonlinear response of the semiconductor medium through the effective third-order optical susceptibility $\chi_e^{(3)}$, which governs the interaction between the pump wave and the generated modulated modes. The hydrodynamic plasma model is particularly appropriate for the present analysis because it simplifies the treatment of carrier dynamics by replacing the microscopic description of individual streaming electrons with an equivalent electron fluid representation characterized by macroscopic quantities such as the average carrier density, mean velocity, and related plasma parameters.

Although this approach provides a convenient and physically meaningful description of the nonlinear carrier response, its applicability is limited to the regime where the wavelength of the perturbation remains sufficiently large compared with the carrier mean free path. Thus, the analysis is valid under the condition $kl \ll 1$, where k represents the wave number of the interacting mode and l denotes the carrier mean free path.

2.1 Effective third-order optical susceptibility

The propagation of a spatially uniform pump wave $\vec{E}_0 = \hat{x}E_0 \exp(-i\omega_0 t)$ (traveling along the x-axis) in a doped III–V semiconductor crystal subjected to a static magnetic field $\vec{B}_0 = \hat{z}B_0$ oriented along the z-axis has been considered. The applied magnetic field is taken to be perpendicular to both the direction of propagation of the pump wave and the associated electric field components.

The incident pump field is assumed to remain spatially uniform ($|\vec{k}_0| \approx 0$) within the interaction region. This approximation is justified under the dipole approximation, where the wavelengths of the excited modulated waves are much smaller than the characteristic spatial variation length of the pump field [29]. Consequently, the spatial variation of the pump amplitude can be neglected, allowing the gradient terms associated with the pump field distribution to be omitted from the analysis.

The underlying physical mechanism of the considered modulational interaction can be described as follows. An intense electromagnetic pump wave propagating through the semiconductor medium induces carrier-density perturbations, which facilitate a deformation potential coupling between the free charge carriers, represented by the electron-plasma wave (plasmon), and the transverse optical phonon (TOP) modes associated with lattice vibrations.

This nonlinear coupling process generates a strong space-charge electric field oscillating at the TOP frequency. The induced field interacts with the modulated wave and drives the transverse optical phonon mode at the corresponding modulation frequency. As a result, energy is transferred from the incident pump wave to the coupled plasmon–TOP system, leading to significant amplification of the modulated electromagnetic wave through nonlinear optical pumping.

The fundamental equations governing the PL–TOP interaction-induced modulational amplification process are given as follows:

$$\frac{\partial \vec{v}_0}{\partial t} + \nu\vec{v}_0 = -\frac{e}{m}[\vec{E}_0 + (\vec{v}_0 \times \vec{B}_0)] = -\frac{e}{m}\vec{E}_e \tag{1}$$

$$\frac{\partial \vec{v}_1}{\partial t} + \nu\vec{v}_1 + \left(\vec{v}_0 \cdot \frac{\partial}{\partial x}\right)\vec{v}_1 = -\frac{e}{m}[\vec{E}_1 + (\vec{v}_1 \times \vec{B}_0)] \tag{2}$$

$$\frac{\partial n_1}{\partial t} + n_0\frac{\partial v_1}{\partial x} + n_1\frac{\partial v_0}{\partial x} + v_0\frac{\partial n_1}{\partial x} = 0 \tag{3}$$

Equations (1) and (2) represent the momentum transfer equations corresponding to the pump wave and the generated product waves, respectively. In these equations, $\vec{v}_0$ and $\vec{v}_1$ denote the oscillatory fluid velocities of the charge carriers induced by the pump electric field and the electric field associated with the product waves, respectively. The parameter ν represents the phenomenological electron–electron collision frequency responsible for momentum relaxation, while m denotes the effective mass of an electron having charge e.

In Eq. (1), the effective electric field $\vec{E}_e$ includes the contribution of the Lorentz force $\vec{v}_0 \times \vec{B}_0$ arising from the externally applied static magnetic field. In Eq. (2), the contribution of the Lorentz force $\vec{v}_0 \times \vec{B}_1$ associated with the perturbed magnetic field of the propagating waves has been neglected by assuming that its effect is insignificant for the present analysis.

It is important to note that the nonlinear optical response of the semiconductor plasma originates from the nonlinear terms present in the carrier motion, particularly the convective derivative $(\vec{v}.\nabla)\vec{v}$ and the Lorentz force term $e(\vec{v}_{0,1} \times \vec{B}_0)$. Both of these contributions depend on the total intensity of the applied electromagnetic radiation and are responsible for generating the nonlinear carrier dynamics leading to modulational interaction.

Equation (3) represents the continuity equation, where n_0 and n_1 correspond to the equilibrium and perturbed electron densities, respectively.

In a weakly polar semiconductor crystal, the scattering of a high-frequency pump wave is significantly influenced by the excitation of normal optical phonon vibrational modes. For the present analysis, the semiconductor medium is considered to consist of *N* harmonic oscillators per unit volume, where each oscillator is characterized by its equilibrium position x, molecular mass *M*, and corresponding normal vibrational coordinate $u(x,t)$.

These harmonic oscillators represent the lattice vibrations associated with the optical phonon modes of the crystal. The interaction between the electromagnetic pump field and these lattice vibrations provides the basis for the nonlinear coupling mechanism responsible for the plasmon–transverse optical phonon interaction-induced modulational amplification.

The equation describing the motion of an individual harmonic oscillator, representing the optical phonon mode of the semiconductor lattice, can be expressed as given in Ref. [30]. This equation governs the dynamical response of the lattice vibration under the influence of the internal restoring force and the external driving field associated with the nonlinear interaction process.

$$\frac{\partial^2 u(x,t)}{\partial t^2} + 2\Gamma_{top}\frac{\partial u(x,t)}{\partial t} + \omega_{top}^2 u(x,t) = \frac{F(x,t)}{M}. \qquad (4)$$

Here, $u(x,t) = u\exp[i(k_{op}x - \omega_{op}t)]$ represents the displacement of the lattice ions from their equilibrium positions due to the influence of interacting electromagnetic fields. The parameter Γ_{top} denotes the damping coefficient, which is considered equivalent to the phenomenological collision frequency of the transverse optical phonons (TOPs), i.e., $\sim 10^{-2}\omega_{top}$. The quantity ω_{top} represents the undamped molecular vibrational frequency and is assumed to correspond to the transverse optical phonon frequency. In the present analysis, the TOP frequency is considered much smaller than the pump-wave frequency ($\omega_{top} \ll \omega_0$).

The driving force per unit volume $F(x,t)$ acting on the lattice oscillator arises from two major contributions ($F = F^{(1)} + F^{(2)}$): the force $F^{(1)} = q_s E_{top}$ associated with the Szigeti effective charge q_s and the force $F^{(2)} = 0.5\varepsilon\alpha_d \bar{E}^2(x,t)$ resulting from the differential polarizability $\alpha_d[= (\partial\alpha/\partial u)_0]$ of the medium. These forces represent the coupling between the lattice vibrations and the electromagnetic fields present within the semiconductor crystal. The term E_{top} denotes the electric field associated with the transverse optical phonon mode.

The overbar notation indicates temporal averaging over several optical cycles, since the molecular lattice cannot follow the rapid oscillations occurring at optical frequencies. $\varepsilon = \varepsilon_0\varepsilon_\infty$. The parameters ε_0 and ε_∞ represent the static (low-frequency) and high-frequency dielectric constants of the semiconductor medium, respectively. These quantities characterize the dielectric response of the crystal and influence the strength of the plasmon–transverse optical phonon coupling.

The redistribution and migration of charge carriers within the semiconductor medium result in charge separation, which generates a strong space-charge electric field E_1. This induced space-charge field plays a significant role in mediating the nonlinear coupling between the plasmon mode and the transverse optical phonon mode.

The magnitude of the space-charge field can be derived by combining the continuity equation [Eq. (3)] with the Poisson's equation [Eq. (5)], leading to the following expression:

$$\frac{\partial E_1}{\partial x} + N\alpha_d \frac{\partial u^*}{\partial x} E_0 = -\frac{n_1 e}{\varepsilon}. \qquad (5)$$

The modulational interaction process considered in the present analysis must satisfy the appropriate phase-matching conditions: $\hbar\vec{k}_\pm = \hbar\vec{k}_0 \pm \hbar\vec{k}_{top}$, and $\hbar\omega_\pm = \hbar\omega_0 \pm \hbar\omega_{top}$, which represent the fundamental requirements for the conservation of momentum and energy, respectively. The momentum conservation condition ensures the proper spatial coupling between the interacting waves, whereas the energy conservation condition governs the resonant exchange of energy among the pump wave, modulated waves, and transverse optical phonon mode.

For a spatially uniform laser excitation, the pump field is assumed to satisfy the condition $|\vec{k}_0| \approx 0$ (or equivalently, $|\vec{k}_\pm| \approx |\vec{k}_0 \pm \vec{k}_{top}| \approx |\vec{k}_1| = k$). Under this approximation, the variation of the pump wave over the interaction region can be neglected, which is consistent with the dipole approximation. Consequently, the phase-matching conditions simplify, allowing efficient coupling between the plasmon mode, transverse optical phonon mode, and the generated sideband waves.

The carrier-density perturbation generated by the intense pump beam is coupled with the transverse optical phonon (TOP) mode and oscillates at the characteristic TOP frequency, ω_{top}. These density fluctuations periodically modulate the incident pump wave, resulting in the generation of forced perturbations at the upper and lower sideband frequencies, represented by $\omega_+ (= \omega_0 + p\omega_{top})$ and $\omega_- (= \omega_0 - p\omega_{top})$, respectively, where the sideband order is denoted by an integer *p*.

In the present analysis, only the resonant sideband frequencies are considered, satisfying the condition (p = 1). This assumption is justified by considering a sufficiently long interaction length, corresponding to an effectively infinite semiconductor crystal, where higher-order scattering processes (p > 1) become off-resonant and contribute negligibly to the nonlinear interaction.

Therefore, the modulational coupling is primarily governed by the interaction among the incident pump wave and the generated upper and lower sideband waves, which are directly coupled with the TOP mode through the nonlinear carrier-density perturbations.

The equation describing the carrier-density fluctuations associated with the coupled electron-plasma wave in a weakly polar, magnetized, n-type doped III–V semiconductor crystal is derived by employing Eqs. (1)–(5) along with the linearized perturbation approach. The derivation is carried out under the collision-dominated regime ($\nu \gg kv_0, \omega$), where the effect of carrier collisions plays a significant role in determining the dynamics of the electron plasma response.

The resulting expression describes the evolution of electron-density perturbations arising from the nonlinear interaction between the electron-plasma mode and the transverse optical phonon excitation in the semiconductor medium.

$$\frac{\partial^2 n_1(\omega_\pm,k_\pm)}{\partial t^2}+\nu\frac{\partial n_1(\omega_\pm,k_\pm)}{\partial t}+\bar{\omega}_p^2 n_1(\omega_\pm,k_\pm)-\frac{i(e/m)(N/M)kn_0E_{top}}{\varepsilon_0(\omega_{top}^2+i\Gamma_{top}\omega_{top})}\left(q_s^2+\frac{\varepsilon^2\alpha_d^2}{4}\right)=-ikn_1(\omega_\pm,k_\pm)\bar{E}\ , \tag{6}$$

where $\bar{E}=-\left(\frac{e}{m}E_0+\omega_0 v_{0y}\right)$, $\bar{\omega}_p^2=\frac{\nu^2\omega_p^2}{(\nu^2+\omega_c^2)}$,

in which $\omega_c=\frac{eB_0}{m}$, and $\omega_p=\left(\frac{n_0e^2}{m\varepsilon}\right)^{1/2}$.

represent the electron-cyclotron frequency and electron-plasma frequency, respectively.

In deriving Eq. (6), the Doppler shift contribution has been neglected by considering the condition ($\omega_0 \gg \nu > kv_0$) that the carrier drift velocity is sufficiently small compared with the phase velocity of the interacting wave. This approximation allows the simplification of the carrier-density perturbation equation by eliminating the frequency shift associated with the motion of charge carriers.

Furthermore, the contribution of the pump electric field component along the *y*-direction and the associated pump magnetic field has been ignored under the assumption ($\omega_p \approx \omega_c \approx \omega_0$) that their effects are negligible in comparison with the dominant interaction terms involved in the present analysis. This approximation simplifies the coupled-mode formulation while retaining the essential physics of the plasmon–transverse optical phonon (PL–TOP) interaction-based modulational amplification process.

The carrier-density perturbations generated at the forced wave frequencies, corresponding to the upper and lower sideband frequencies, can be expressed in the following mathematical forms. These perturbations represent the oscillatory response of the electron plasma system induced by the nonlinear coupling between the pump field and the transverse optical phonon mode.

The respective expressions for the density fluctuations associated with the upper and lower sidebands are given as:

$$n_1(\omega_+,k_+)=\frac{i(e/m)(N/M)kn_0(Y_+)^{-1}E_{top}}{\varepsilon\Omega_{od}^2}\left(q_s^2+\frac{\varepsilon^2\alpha_d^2}{4}\right), \tag{7a}$$

and

$$n_1(\omega_-,k_-)=\frac{i(e/m)(N/M)kn_0(Y_-)^{-1}E_{top}}{\varepsilon\Omega_{od}^2}\left(q_s^2+\frac{\varepsilon^2\alpha_d^2}{4}\right). \tag{7b}$$

Here, the term $\Omega_{od}^2=\omega_{op}^2+i\Gamma_{op}\omega_{op}$ represents the dispersion relation of the transverse optical phonon mode. The quantities

$Y_+=\bar{\omega}_p^2-\omega_+^2-i\nu\omega_++ik\bar{E}$, and

$Y_-=\bar{\omega}_p^2-\omega_-^2-i\nu\omega_-+ik\bar{E}$, in which

$\omega_+=\omega_{top}+\omega_0$ and $\omega_-=\omega_{top}-\omega_0$ correspond to the frequencies of the upper and lower sideband waves, respectively, generated due to the modulation of the incident pump field.

While deriving Eqs. (7a) and (7b), it has been assumed that the electron-density perturbations associated with the sideband frequencies $n_1(\omega_\pm,k_\pm)$ exhibit a harmonic variation of the form $\exp[i(\omega_\pm t-k_\pm x)]$. The obtained expressions clearly demonstrate that the upper and lower sideband waves are nonlinearly coupled with the TOP mode through the induced carrier-density perturbations in the presence of an intense pump field.

Furthermore, these equations indicate that the magnitude of the density fluctuations $n_1(\omega_\pm,k_\pm)$ depends strongly on the pump-field amplitude through the coupling parameter $\bar{E}$. The density perturbations generated at the sideband frequencies modify the electromagnetic response of the semiconductor medium and consequently influence the propagation characteristics of the generated waves. These effects can be analyzed further by applying the general electromagnetic wave equation to the coupled system.

In the present analysis of modulational interaction in a magnetized semiconductor crystal, the contribution of the transition dipole moment has been neglected in order to focus exclusively on the role of the nonlinear induced current density in determining the polarization response of the semiconductor medium.

The nonlinear current density generated at the upper and lower sideband frequencies arises due to the nonlinear motion of charge carriers under the combined influence of the pump electromagnetic field, induced space-charge field, and externally applied magnetic field. These nonlinear current components contribute directly to the induced polarization of the medium and govern the effective third-order nonlinear optical response associated with the PL–TOP interaction.

The expressions for the induced nonlinear current densities corresponding to the upper and lower sidebands are given by:

$$J(\omega_+)=-en_1(\omega_+)v_0 \tag{8a}$$

and

$$J(\omega_-)=-en_1(\omega_-)v_0^*\ . \tag{8b}$$

Henceforth, the induced polarization $P(\omega_\pm)$ of the semiconductor medium at the modulated frequencies is obtained by treating it as the time integral of the corresponding nonlinear current density $J(\omega_\pm)$. This approach relates the nonlinear carrier response directly to the induced polarization and enables the evaluation of the effective nonlinear optical susceptibility of the medium.

Thus, we have:

$$P(\omega_\pm) = \int J(\omega_\pm)dt \,. \tag{9}$$

The effective nonlinear polarization of the modulated wave is obtained by defining the nonlinear contribution of the induced polarization arising from the interaction between the pump field, carrier-density perturbations, and the transverse optical phonon mode. This effective polarization represents the nonlinear optical response of the semiconductor medium and provides the basis for evaluating the third-order optical susceptibility associated with the PL–TOP interaction.

Thus, the effective nonlinear polarization of the modulated wave can be expressed as:

$$P_e = P(\omega_+) + P(\omega_-) \,. \tag{10}$$

For efficient amplification of the modulated waves, it is necessary that the upper and lower sidebands contribute equally to the nonlinear interaction process. The combined effect of these sidebands transfers the modulation energy to the transverse optical phonon (TOP) mode, resulting in its amplification through the nonlinear coupling mechanism.

Therefore, by combining Eqs. (7)–(10), the total effective nonlinear polarization of the medium can be obtained as:

$$P_e = \frac{i(e/m)^2 (N/M) k \omega_p^2 \omega_0 (Y_1) |E_0| |E_{top}|}{\Omega_{od}^2 (\omega_0^2 - \omega_c^2)} \left(q_s^2 + \frac{\varepsilon^2 \alpha_d^2}{4} \right), \tag{11}$$

where

$$Y_1 = \frac{1}{\omega_+} (\Omega_+^2 - i\nu\omega_+ + ik\bar{E})^{-1} - \frac{1}{\omega_-} (\Omega_-^2 - i\nu\omega_- + ik\bar{E})^{-1} ,$$

in which $\Omega_+^2 = \bar{\omega}_p^2 - \omega_+^2$, and $\Omega_-^2 = \bar{\omega}_p^2 - \omega_-^2$.

Equation (11) demonstrates that the effective nonlinear polarization P_e establishes the coupling between the perturbed carrier-density waves oscillating at the sideband frequencies $\omega_\pm$ and the transverse optical phonon (TOP) mode oscillating at the frequency ω_{top}.

By applying suitable algebraic simplifications to Eq. (11), the expression for the third-order nonlinear polarization $P_e^{(3)}$ associated with the PL–TOP interaction is obtained as:

$$P_e^{(3)} = \frac{2(e/m)^2 (N/M) k^2 \omega_p^2 (\Omega^2 - \nu^2)(Y_2)^{-1} |E_0|^2 |E_{top}|}{\Omega_{od}^2 (\omega_0^2 - \omega_c^2)^2} \left(q_s^2 + \frac{\varepsilon^2 \alpha_d^2}{4} \right), \tag{12}$$

where $Y_2 = \left(\Omega^2 + \nu^2 - \frac{k^2 \bar{E}^2}{\omega_0^2} \right)^2 + \frac{4k^2 \Omega^2 \bar{E}^2}{\omega_0^2}$, in which $\Omega^2 = \omega_0^2 - \bar{\omega}_p^2$.

The transverse components of the oscillatory electron-fluid velocity $\vec{v}_0$ under the combined influence of the pump electric field $\vec{E}_0$ and the externally applied static magnetic field $\vec{B}_0$ can be obtained from the electron momentum transfer equation [Eq. (1)].

By resolving Eq. (1) into transverse components and considering the effect of the Lorentz force due to the applied magnetic field, the corresponding velocity components are expressed as:

$$v_{0x} = \frac{\bar{E}}{\nu - i\omega_0} \quad \text{and} \quad v_{0y} = \frac{-(e/m) E_0 \omega_c}{(\omega_0^2 - \omega_c^2)} \,. \tag{13}$$

By defining the induced effective third-order nonlinear polarization in terms of the generated modulated electric field ($P_e^{(3)} = \varepsilon_0 \chi_e^{(3)} |E_0|^2 |E_{top}|$) and using the expression obtained in Eq. (12), the nonlinear optical response of the semiconductor medium can be characterized through the complex effective third-order susceptibility $\chi_e^{(3)}$.

The effective susceptibility incorporates both the dispersive and absorptive contributions arising from the nonlinear interaction between the plasmon mode and the transverse optical phonon mode under the influence of the externally applied magnetic field. At the modulated frequencies $\omega_\pm$, the complex third-order susceptibility is therefore obtained as:

$$\chi_e^{(3)} = \frac{2(e/m)^2 (N/M) k^2 \omega_p^2 (\Omega^2 - \nu^2)(Y_2)^{-1}}{\varepsilon_0 \Omega_{od}^2 (\omega_0^2 - \omega_c^2)^2} \left(q_s^2 + \frac{\varepsilon^2 \alpha_d^2}{4} \right). \tag{14}$$

It is evident from Eq. (14) that the effective third-order susceptibility $\chi_e^{(3)}$ is a complex quantity, representing both the dispersive and absorptive responses of the semiconductor medium. Therefore, it can be separated into its real and imaginary components.

The real part of the susceptibility corresponds to the nonlinear dispersive response of the medium, whereas the imaginary part represents the nonlinear absorption or gain characteristics associated with the modulational interaction.

Thus, the complex third-order susceptibility may be expressed in terms of its real and imaginary parts as:

$$[\chi_e^{(3)}] = [\chi_e^{(3)}]_r + i[\chi_e^{(3)}]_i \,, \tag{15a}$$

where

$$[\chi_e^{(3)}]_r = \frac{2(e/m)^2 (N/M) k^2 \omega_p^2 \omega_{top}^2 (\Omega^2 - \nu^2)(Y_2)^{-1}}{\varepsilon_0 (\omega_{top}^4 + \Gamma_{top}^2 \omega_{top}^2)(\omega_0^2 - \omega_c^2)^2} \left(q_s^2 + \frac{\varepsilon^2 \alpha_d^2}{4} \right) \tag{15b}$$

and

$$[\chi_e^{(3)}]_i = \frac{-2(e/m)^2 (N/M) k^2 \omega_p^2 \omega_{top} \Gamma_{top} (\Omega^2 - \nu^2)(Y_2)^{-1}}{\varepsilon_0 (\omega_{top}^4 + \Gamma_{top}^2 \omega_{top}^2)(\omega_0^2 - \omega_c^2)^2} \left(q_s^2 + \frac{\varepsilon^2 \alpha_d^2}{4} \right). \tag{15c}$$

Here, the subscripts "*r*" and "*i*" associated with the susceptibility denote its real and imaginary components, respectively. The above analysis has been carried out in the highly doped regime, i.e. for $\omega_p \approx \omega_0 (\approx \omega_\pm)$ while $\omega_p \gg \nu, \omega_{top}$, where the electron concentration is sufficiently large and the corresponding conditions for strong plasmon–TOP coupling are satisfied.

Equations (15b) and (15c) describe the steady-state optical response of the semiconductor medium in the presence of a transverse magnetic field and determine the nonlinear propagation characteristics of the modulated waves. The formulation clearly indicates that the effective susceptibility of the crystal is strongly influenced by the equilibrium carrier concentration n_0 through the electron-plasma frequency ω_p and by the externally applied d.c. magnetic field B_0 through the electron-cyclotron frequency ω_c.

The real part of the third-order susceptibility $[\chi_e^{(3)}]_r$ [Eq. (15b)] governs the nonlinear dispersive properties of the medium, whereas the imaginary part $[\chi_e^{(3)}]_i$ [Eq. (15c)] determines the steady-state growth rate of the propagating modulated wave. These components therefore provide essential information regarding the dispersion and amplification characteristics of the PL–TOP interaction-based modulational process.

From Eq. (15b), it can be inferred that the real part of the effective third-order susceptibility $[\chi_e^{(3)}]_r$ can acquire either positive or negative values, depending on the factor $(\Omega^2 - \nu^2)$, i.e. relative values of Ω (and hence ω_p) and ν.

A positive dispersion of the modulated wave occurs under the condition $\omega_p \gg \nu$, where this factor becomes positive. In this regime, the nonlinear refractive index decreases with increasing distance from the beam axis. According to Snell's law, this spatial variation in refractive index causes the velocity of the modulated wave to increase away from the beam axis, resulting in the phenomenon of self-focusing.

Conversely, negative dispersion occurs when the relevant factor becomes negative. In this case, the nonlinear refractive index increases with distance from the beam axis, leading to a decrease in wave velocity away from the axis and resulting in the occurrence of self-defocusing of the modulated wave.

To investigate the possibility of modulational amplification in the semiconductor medium, the relation given in Ref. [21] is employed. This relation provides a direct connection between the nonlinear optical response of the medium and the growth characteristics of the modulated wave, enabling the evaluation of the conditions required for amplification.

$$\alpha_e = \frac{k}{2\varepsilon_1} [\chi_e^{(3)}]_i |E_0|^2, \tag{16}$$

where α_e represents the effective nonlinear absorption coefficient of the semiconductor medium. The nonlinear amplification of the modulated signal occurs only when the effective nonlinear absorption coefficient obtained from Eq. (16) becomes negative.

Using Eqs. (15c) and (16), it can be inferred that a positive growth rate $g (= |-\alpha_e|)$ of the modulated wave requires the imaginary part of the effective third-order susceptibility $[\chi_e^{(3)}]_i$ to be negative. This condition ensures that the nonlinear gain dominates over the absorption losses, resulting in the amplification of the modulated signal.

Therefore, the conditions required for achieving a positive growth rate can be classified into two cases:

Case (i): When the imaginary part of the third-order susceptibility and the effective nonlinear absorption coefficient satisfy the condition corresponding to negative nonlinear absorption, i.e. $\nu > \Omega$ and $\omega_0 < \omega_c$.

Case (ii): When the alternate interaction regime between the electron-plasma mode and transverse optical phonon mode satisfies the condition for negative nonlinear absorption, resulting in positive growth of the modulated wave, i.e. $\nu < \Omega$ and $\omega_0 > \omega_c$.

In the first case, it may be inferred that the presence of electron–electron collisions in conjunction with an external magnetic field is not an essential prerequisite for the onset of instability. However, modulational growth of the signal wave can be achieved by applying an external magnetic field for which the electron-cyclotron frequency exceeds the pump-wave frequency, i.e. $\omega_c > \omega_0$.

Under this condition, the magnetic-field-induced modification of carrier dynamics enhances the nonlinear coupling between the plasmon and transverse optical phonon modes, thereby promoting the growth of the modulated signal wave in the semiconductor medium.

In the second case, modulational growth in the semiconductor medium can be achieved by applying an external magnetic field for which the electron-cyclotron frequency is lower than the pump-wave frequency, i.e. $\omega_c < \omega_0$. This indicates that both electron–electron collisions and the externally applied magnetic field play important roles in modifying the modulational instability characteristics of the system.

However, when the electron-cyclotron frequency becomes comparable to or exceeds the pump-wave frequency, the effect of cyclotron absorption may dominate the instability process. Moreover, achieving such a condition is relatively difficult under practical experimental conditions. Therefore, in the present investigation of modulational amplification in semiconductor media, attention is restricted to the latter case, as it can be readily realized under normal operating conditions.

2.2 Threshold pump field for the onset of PL-TOP interaction based modulational amplification

As is well established, modulational amplification occurs only when the pump-field amplitude exceeds a certain critical threshold value. This threshold condition represents the minimum pump intensity required to overcome the damping effects and initiate the instability growth of modulated wave.

The threshold pump amplitude can be determined by setting the growth rate of the modulated wave equal to zero, i.e., by applying the condition for the onset of instability ($[\chi_e^{(3)}]_i = 0$). Under this limiting condition, the expression for the threshold pump amplitude is obtained as:

$$\left|E_{0,th}\right|_{pl-top} = \frac{\Omega(\omega_0^2 - \omega_c^2)}{(e/m)k\omega_0^2}. \tag{17}$$

From Eq. (17), it can be concluded that the modulational instability of the signal wave possesses a finite pump-amplitude threshold, even in the absence of damping mechanisms. This indicates that a minimum pump-field strength is required to initiate the PL–TOP interaction-based modulational amplification process. The threshold pump amplitude $(E_{0,th})_{pl-top}$ for the onset of modulational amplification exhibits a complex dependence on the system parameters. It is found to be independent of the effective charge q_s and differential polarizability α_d, which characterize the optical phonon response of the crystal. However, it shows a strong dependence on the wave number k, equilibrium carrier concentration k (through the electron-plasma frequency ω_p and the associated parameter Ω), and the externally applied d.c. magnetic field B_0 (through the electron-cyclotron frequency ω_c).

Thus, the threshold condition for PL–TOP interaction-based modulational amplification is primarily governed by the electronic properties of the semiconductor medium and the externally controlled magnetic-field-induced carrier dynamics.

2.3 Growth rate of PL-TOP interaction based modulationally amplified wave

The growth rate of the PL–TOP interaction-based modulationally amplified wave for pump-field amplitudes significantly higher than the threshold electric field can be evaluated using the imaginary part of the effective third-order susceptibility [Eq. (15c)] and the relation given in Eq. (16).

By substituting the nonlinear response parameters obtained from Eqs. (15c) and (16), the expression for the growth rate of the amplified modulated wave is obtained as:

$$(g)_{pl-top} = \frac{-(e/m)^2 (N/M) k^3 \omega_p^2 \omega_{top} \Gamma_{top} (\Omega^2 - \nu^2)(Y_2)^{-1} \left|E_0\right|^2}{\varepsilon_0 \varepsilon_1 (\omega_{top}^4 + \Gamma_{top}^2 \omega_{top}^2)(\omega_0^2 - \omega_c^2)^2} \left(q^2 + \frac{\varepsilon^2 \alpha_d^2}{4}\right) \tag{18}$$

The above formulation reveals that the growth rate $(g)_{pl-top}$ of the PL–TOP interaction-based modulationally amplified signal is independent of the signal-wave frequency $\omega_\pm$. Instead, it is primarily governed by the pump-wave frequency ω_0 and the characteristic frequency of the transverse optical phonon (TOP) mode ω_{top}. This observation is in good agreement with the reported experimental results [31].

Furthermore, the growth rate of the PL–TOP interaction-based modulationally amplified wave is significantly influenced by several important parameters of the semiconductor system, including the wave number k, equilibrium carrier concentration n_0 (through the electron-plasma frequency ω_p and the associated parameter Ω), and the externally applied d.c. magnetic field B_0 (through the electron-cyclotron frequency ω_c). These parameters play a crucial role in controlling the nonlinear coupling strength between the plasmon and TOP modes and thereby determine the amplification characteristics of the modulated wave.

3. Results and discussion

To establish the validity of the present theoretical model, a weakly-polar III–V semiconductor crystal has been selected as the medium of investigation at a temperature of 77 K. The crystal is assumed to be irradiated by a pulsed 10.6 μm CO_2 laser. At this temperature, the absorption coefficient of the semiconductor sample is considerably low in the vicinity of the 10 μm wavelength region. Therefore, the contribution arising from the band-to-band transition mechanism can be safely neglected.

Under these conditions, the optical response of the semiconductor is predominantly governed by the free-carrier-induced nonlinear effects, making the selected parameters suitable for analyzing the PL–TOP interaction-based modulational amplification process.

The physical constants involved are [32]:

$m = 0.014 m_0$; where m_0 is the free electron mass, $M = 236.7$, $N = 1.48\times10^{28}$ m^{-3}, $q_s = 3.2\times10^{-20}$ C, $\alpha_d = 1.68\times10^{16}$ SI units, $\Gamma_{top} = 3.7\times10^{11}$ s^{-1}, $\omega_{top} = 3.64\times10^{13}$ s^{-1}, $\nu = 3.5\times10^{11}$ s^{-1}, $\varepsilon_1 = 17.72$, and $\varepsilon_\infty = 15.68$.

This set of data is related to a typical n-InSb crystal, however, the results obtained in previous section may be applied to any III-V semiconductor crystal.

3.1 Threshold characteristics for the onset of PL-TOP interaction based modulational amplification

Using the physical parameters corresponding to n-InSb given above, the dependence of the threshold pump amplitude $(E_{0,th})_{pl-top}$ required for the onset of PL–TOP interaction-based modulational amplification can be investigated from Eq. (17). The influence of important controlling parameters, namely the wave number k, the externally applied magnetic field B_0 (expressed through the electron-cyclotron frequency ω_c), and the doping concentration n_0 (represented through the electron-plasma frequency ω_p), has been analyzed.

The numerical results obtained from Eq. (17) are presented in Figs. 1–3, which illustrate the variation of the threshold pump amplitude with these parameters and provide insight into the conditions required for initiating modulational amplification in the semiconductor medium.

Fig. 1 illustrates the variation of the threshold pump amplitude with the wave number at a fixed set of operating conditions ($\omega_c, \omega_p \approx \omega_0$). It can be observed that the threshold pump amplitude is relatively higher for smaller values of the wave number. As the magnitude of the wave number increases, the threshold pump amplitude exhibits a parabolic decrease.

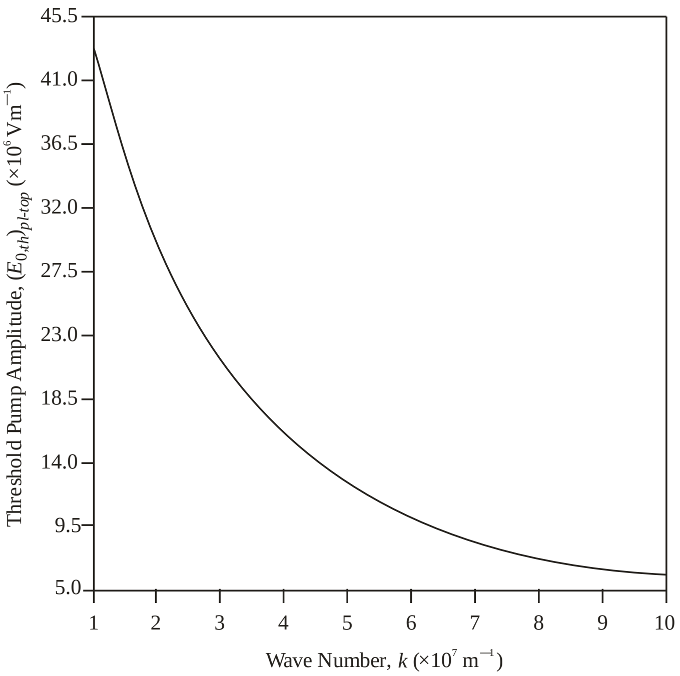

Figure 1: Variation of threshold pump amplitude $(E_{0,th})_{pl-top}$ for the onset of PL-TOP interaction based modulational amplification with wave number k at $\omega_c, \omega_p \approx \omega_0$.

This behavior can be attributed to the dependence of the threshold condition on the wave-number-dependent coupling term $(E_{0,th})_{pl-top} \propto k^{-1}$, as evident from Eq. (17). The increase in wave number enhances the nonlinear interaction strength between the coupled modes, thereby reducing the minimum pump-field amplitude required to initiate the PL–TOP interaction-based modulational amplification.

Fig. 2 presents the variation of the threshold pump amplitude $(E_{0,th})_{pl-top}$ with the externally applied magnetic field B_0, represented through the electron-cyclotron frequency ω_c / ω_0, at the specified operating conditions ($\omega_p \approx \omega_0$). It is observed that the threshold pump amplitude remains almost independent of the magnetic field in the low-field regime ($\omega_c < \omega_0$), where the electron-cyclotron frequency is much smaller than the pump-wave frequency.

With an increase in the magnetic field, the threshold pump amplitude decreases sharply and reaches a minimum value when the electron-cyclotron frequency becomes comparable to the pump-wave frequency ($\omega_c \sim \omega_0$). For the n-InSb crystal irradiated by a pulsed CO_2 laser, this minimum corresponds to a specific magnetic-field value and indicates the optimum condition for enhancing the PL–TOP interaction.

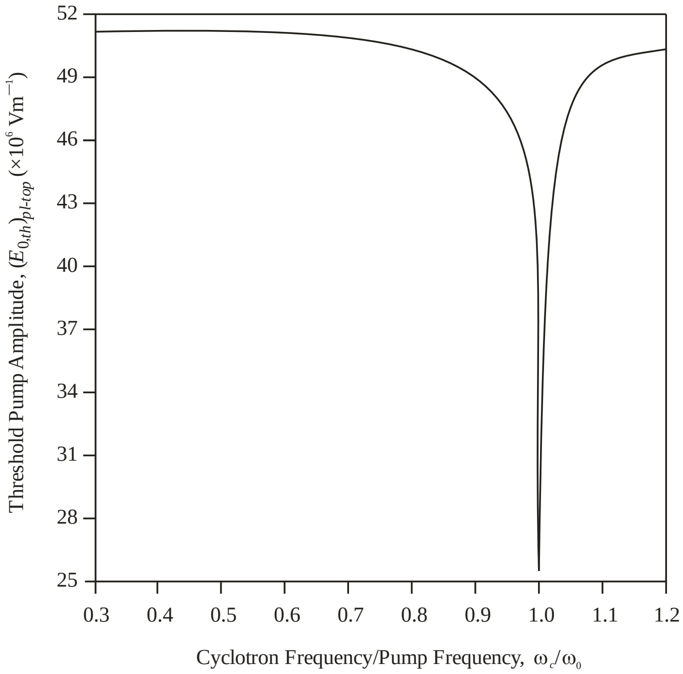

Figure 2: Variation of threshold pump amplitude $(E_{0,th})_{pl-top}$ for the onset of PL-TOP interaction based modulational amplification with externally applied magnetic field B_0 (via ω_c / ω_0) at $\omega_p \approx \omega_0$ and $k = 2.5 \times 10^7$ m^{-1}.

The occurrence of this minimum can be attributed to the resonant enhancement of the nonlinear coupling between the plasmon and transverse optical phonon modes, i.e. $(E_{0,th})_{pl-top} \propto (\omega_0^2 - \omega_c^2)$, as evident from Eq. (17). However, with further increase in the magnetic field, the threshold pump amplitude increases sharply and eventually becomes almost independent of the magnetic field in the high-field regime, where the electron-cyclotron frequency exceeds the pump-wave frequency ($\omega_c > \omega_0$).

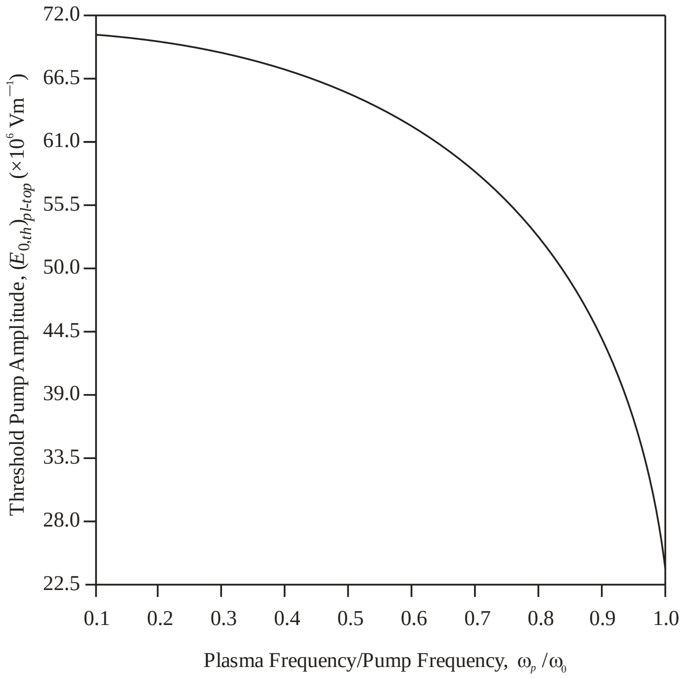

Figure 3: Variation of threshold pump amplitude $(E_{0,th})_{pl-top}$ for the onset of PL-TOP interaction based modulational amplification with doping concentration n_0 (via ω_p / ω_0) at $\omega_c \approx \omega_0$, $k = 2.5 \times 10^7$ m^{-1}.

Fig. 3 depicts the variation of the threshold pump amplitude $(E_{0,th})_{pl-top}$ with the doping concentration n_0 (via ω_p/ω_0), at the specified operating conditions ($\omega_c \approx \omega_0$). It can be observed that the threshold pump amplitude decreases with increasing doping concentration.

For lower doping concentrations ($\omega_p < \omega_0$), the rate of reduction of the threshold pump amplitude with increasing carrier concentration is relatively small. However, as the doping concentration increases, the rate of decrease becomes more pronounced. The threshold pump amplitude reaches a minimum value at a particular doping concentration for the n-InSb crystal irradiated by the pulsed CO_2 laser. This minimum corresponds to the condition ($\omega_p \sim \omega_0$) where the electron-plasma frequency becomes comparable to the pump-wave frequency.

The occurrence of this minimum can be attributed to the enhanced nonlinear coupling between the plasmon and transverse optical phonon modes under the resonance-like condition ($(E_{0,th})_{pl-top} \propto \Omega[=(\omega_0^2 - \bar{\omega}_p^2)^{1/2}]$), as evident from Eq. (17). Beyond this optimum doping level, further increase in carrier concentration results in an increase in the threshold pump amplitude.

3.2 Growth rate of PL-TOP interaction based modulationally amplified wave

Using the material parameters corresponding to n-InSb given above, the dependence of the growth rate $(g)_{pl-top}$ of the PL–TOP interaction-based modulationally amplified wave can be investigated from Eq. (18). The effects of various important parameters, namely the wave number k, the externally applied magnetic field B_0 (expressed through the electron-cyclotron frequency ω_c), the doping concentration n_0 (represented through the electron-plasma frequency ω_p), and the pump-field amplitude E_0 (considered well above the threshold pump amplitude), have been analyzed.

The numerical results obtained from Eq. (18) are presented in Figs. 4–7, which demonstrate the influence of these parameters on the amplification characteristics of the modulated wave. These results provide insight into the conditions required for achieving enhanced modulational growth in weakly-polar magnetoactive doped III–V semiconductor crystals.

Fig. 4 illustrates the variation of the growth rate $(g)_{pl-top}$ of the PL–TOP interaction-based modulationally amplified wave with the wave number k at the specified pump-field amplitude $E_0 = 2.5\times10^8$ Vm^{-1} and operating conditions ($\omega_c, \omega_p \approx \omega_0$). It is observed that the growth rate exhibits the characteristic dependence on the wave vector expected for modulational instability.

For smaller values of the wave number k ($\le 6\times10^7$ m^{-1}), the growth rate increases gradually and exhibits an approximately linear dependence on the wave number. As the magnitude of the wave number is increased further, the growth rate shows a more rapid enhancement and follows a quadratic dependence on the wave number. This behavior indicates that the strength of the nonlinear coupling responsible for the amplification process increases with increasing wave-vector magnitude. The observed variation of the temporal growth rate with wave number is in accordance with the conventional dependence predicted for modulational instability processes.

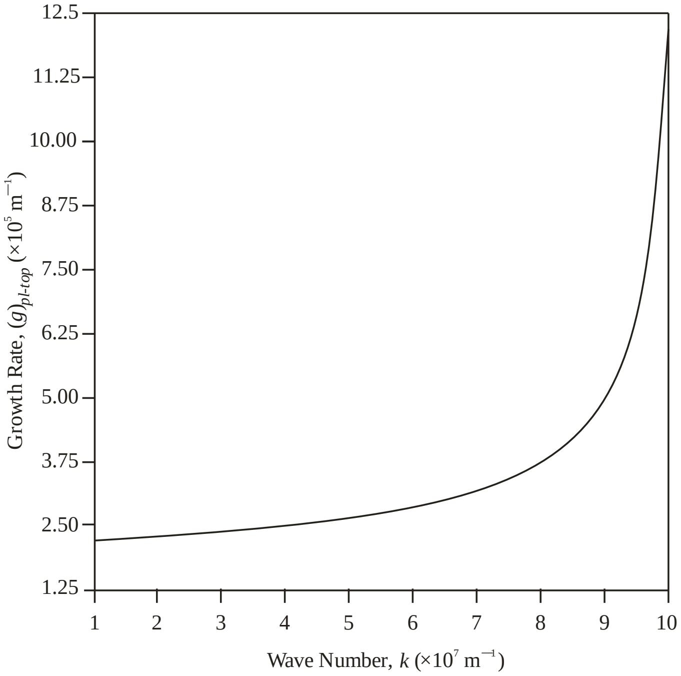

Figure 4: Variation of growth rate $(g)_{pl-top}$ of PL-TOP interaction based modulationally amplified wave with wave number $(E_{0,th})_{pl-top}$ at $\omega_c, \omega_p \approx \omega_0$ and $E_0 = 2.5\times10^8$ Vm^{-1}.

Fig. 5 presents the variation of the growth rate $(g)_{pl-top}$ of the PL–TOP interaction-based modulationally amplified wave with the externally applied magnetic field B_0 (via ω_c/ω_0) at $\omega_p \approx \omega_0$ and the specified pump-field amplitude $E_0 = 2.5\times10^8$ Vm^{-1}.

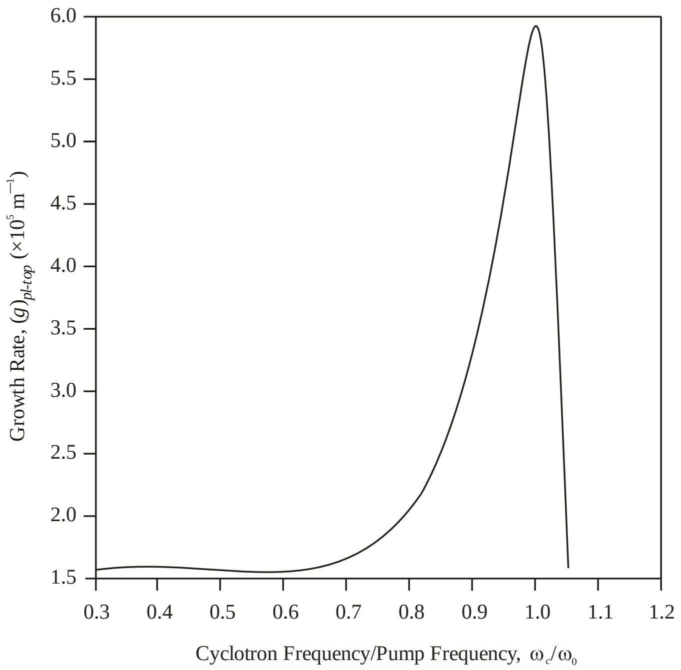

Figure 5: Variation of growth rate $(g)_{pl-top}$ of PL-TOP interaction based modulationally amplified wave with externally applied magnetic field B_0 (via ω_c/ω_0) at $\omega_p \approx \omega_0$, $k = 2.5\times10^7$ m^{-1} and $E_0 = 2.5\times10^8$ Vm^{-1}.

It is observed that the growth rate initially remains very small and is almost independent of the applied magnetic field up to a certain range ($\omega_c = 0.7\omega_0$) of electron-cyclotron frequency. With further increase in the magnetic field, the growth rate increases rapidly due to enhanced coupling between the plasmon and transverse optical phonon modes. It reaches a maximum value when the electron-cyclotron frequency becomes comparable to the pump-wave frequency ($\omega_c \approx \omega_0$), indicating an optimum magnetic-field condition for achieving maximum modulational amplification.

Beyond this optimum value, the growth rate decreases sharply and eventually approaches zero at higher magnetic fields. The occurrence of the peak in the growth rate can be attributed to the enhanced nonlinear interaction term ($(g)_{pl-top} \propto (\omega_0^2 - \omega_c^2)^{-2}$) in Eq. (18), whereas the subsequent reduction is associated with the increase in the magnetic-field-dependent denominator term in the growth-rate expression.

The disappearance of the growth rate beyond a critical magnetic-field value ($\omega_c = 1.05\omega_c$) can be attributed to the dominance of the cyclotron absorption process, which suppresses the instability mechanism. Thus, the finite magnetic-field range over which significant amplification occurs provides a favorable operating window for controlling the modulational instability. This tunable magnetic-field dependence of the growth rate may be effectively utilized in the development of optical switching devices based on magnetoactive semiconductor media.

Fig. 6 illustrates the variation of the growth rate $(g)_{pl-top}$ of the PL–TOP interaction-based modulationally amplified wave with the doping concentration n_0 (via ω_p/ω_0) at $\omega_c \approx \omega_0$ and the specified wave number and pump-field amplitude $E_0 = 2.5\times10^8$ Vm-1.

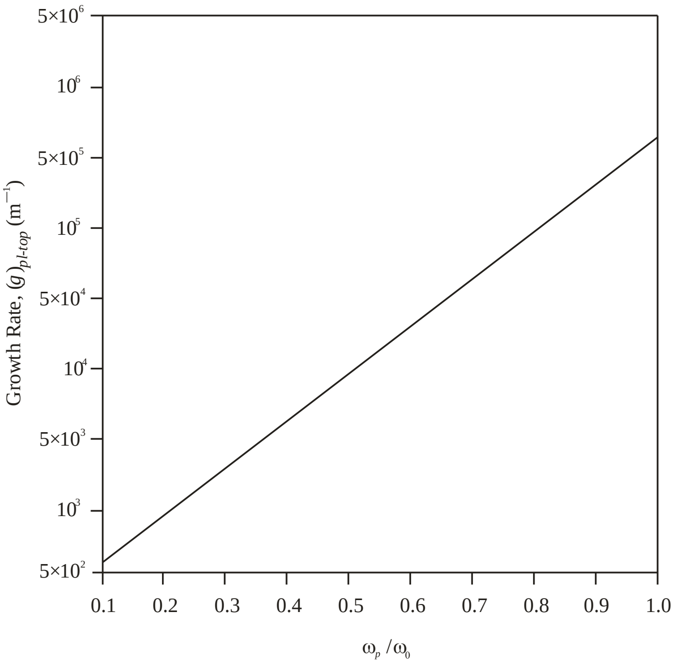

Figure 6: Variation of growth rate $(g)_{pl-top}$ of PL-TOP interaction based modulationally amplified wave with doping concentration n_0 (via ω_p/ω_0) at $\omega_c \approx \omega_0$, $k = 2.5\times10^7$ m-1 and $E_0 = 2.5\times10^8$ Vm-1.

It is observed that the growth rate is relatively small ($\approx 10^2$ m-1) for low doping concentrations ($\omega_p \approx 0.1\omega_c$, corresponding $n_0 = 2.2\times10^{22}$ m-3), corresponding to low equilibrium electron densities. As the doping concentration increases, the growth rate increases almost linearly with the electron density of the semiconductor crystal. At sufficiently high doping concentrations ($\omega_p \sim \omega_0$, corresponding $n_0 = 2.2\times10^{24}$ m-3), the growth rate attains considerably larger values ($\approx 10^6$ m-1), indicating that the efficiency of the modulational amplification process is strongly governed by the free-carrier concentration.

The observed dependence of the growth rate on doping concentration is consistent with the results reported by Salimullah and Singh [33], who investigated the interaction of an extraordinary electromagnetic mode subjected to perturbations parallel to the external magnetic field. The increase in carrier concentration enhances the electron-plasma response and strengthens the nonlinear coupling between the plasmon and transverse optical phonon modes, thereby resulting in a higher growth rate of the modulated wave.

However, the doping concentration should remain below the critical limit at which the electron-plasma frequency exceeds the incident pump-wave frequency. Beyond this limit ($\omega_p > \omega_0$), the semiconductor behaves as an overdense plasma, causing the incident electromagnetic pump wave to be reflected from the medium rather than propagating through it. Consequently, the modulational amplification process cannot be sustained under such conditions.

These results clearly demonstrate that heavily doped III–V semiconductor crystals provide highly favorable conditions for achieving efficient PL–TOP interaction-based modulational amplification and are therefore promising host materials for nonlinear optical and modulational instability phenomena.

The preceding analysis has been carried out for weakly-polar III–V semiconductor crystals, such as n-InSb, with doping concentrations approaching the critical carrier density, i.e., concentrations for which the corresponding electron-plasma frequency becomes comparable to the incident pump-wave frequency **(** $\omega_p \approx \omega_0$). Such heavily doped conditions ensure strong coupling between the free-carrier plasma and the transverse optical phonon mode, thereby facilitating efficient nonlinear interactions and modulational amplification.

Doping concentrations of this magnitude are well within the practical range for III–V semiconductor materials and have been extensively employed in both theoretical and experimental investigations to explore their optical, electrical, and transport properties [34–36]. Therefore, the assumptions adopted in the present analysis are physically realistic and provide a suitable framework for investigating PL–TOP interaction-based modulational amplification in magnetoactive semiconductor media.

Fig. 7 illustrates the variation of the growth rate $(g)_{pl-top}$ of the PL–TOP interaction-based modulationally amplified wave with the pump-field amplitude E_0 at the specified operating conditions ($\omega_c, \omega_p \approx \omega_0$). It is observed that the growth rate initially increases almost linearly with the pump-field amplitude, indicating that a stronger optical pump enhances the nonlinear coupling between the plasmon and transverse optical phonon modes.

As the pump-field amplitude is increased further, the growth rate reaches a maximum value at an optimum pump-field amplitude $E_0 = 3.3\times10^8$ Vm-1. Beyond this optimum value, the growth rate decreases parabolically with further increase in the pump-field amplitude. This reduction may be attributed to the increased influence of higher-order nonlinear effects and the nonlinear saturation embodied in the growth-rate expression.

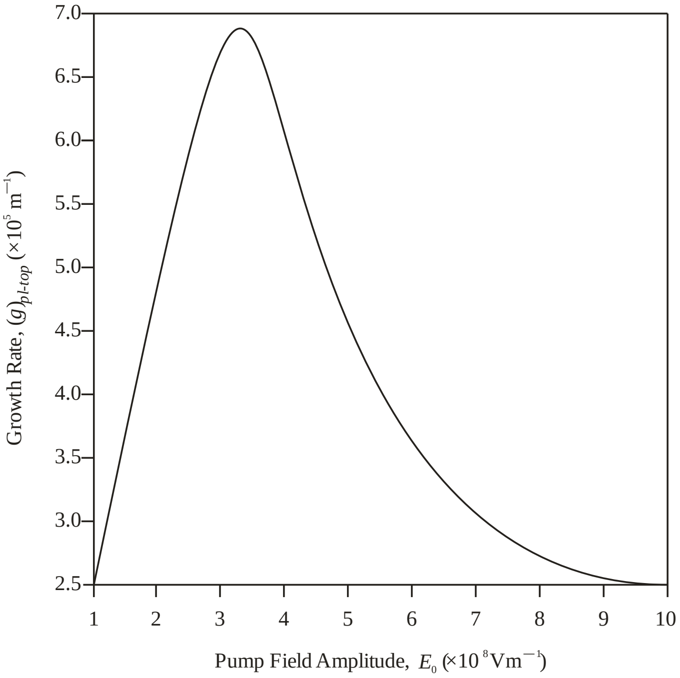

Figure 7: Variation of growth rate $(g)_{pl-top}$ of PL-TOP interaction based modulationally amplified wave with pump field amplitude E_0 at $\omega_c, \omega_p \approx \omega_0$ and $k = 2.5 \times 10^7$ m^{-1}.

The existence of an optimum pump-field amplitude demonstrates that maximum modulational amplification can be achieved by appropriately selecting the pump intensity. Therefore, proper optimization of the pump-field amplitude provides an effective means of maximizing the growth rate of the PL–TOP interaction-based modulationally amplified wave in weakly-polar III–V semiconductor crystals.

4. Conclusions

In the present work, a comprehensive theoretical and numerical investigation has been carried out on the threshold pump field required for the onset of plasmon–transverse optical phonon (PL–TOP) interaction-based modulational amplification and the growth rate of the modulated wave in weakly-polar magnetoactive doped III–V semiconductor crystals. The analysis, based on the hydrodynamic model of semiconductor plasma and the coupled-mode approach, leads to the following conclusions:

1. The hydrodynamic model has been successfully employed to investigate the influence of key physical parameters, including the externally applied transverse magnetostatic field, carrier doping concentration, wave number, and pump-field amplitude, on the threshold pump field and growth characteristics of the modulated wave in weakly-polar magnetoactive III–V semiconductor crystals irradiated by slightly off-resonant, moderate-power pulsed lasers.
2. The threshold pump amplitude required for the onset of PL–TOP interaction-based modulational amplification decreases with increasing wave number. Furthermore, the application of a transverse magnetic field, for which the electron-cyclotron frequency is comparable to the pump-wave frequency, and an increase in doping concentration, for which the electron-plasma frequency approaches the pump-wave frequency, significantly reduce the threshold pump field required to initiate modulational amplification.
3. The growth rate of the PL–TOP interaction-based modulationally amplified wave increases with the wave number of the transverse optical phonon mode. It is further enhanced by the application of an external transverse magnetic field and by increasing the carrier concentration of the semiconductor. Thus, heavily doped semiconductor crystals subjected to an appropriately chosen magnetic field provide favorable conditions for achieving efficient modulational amplification.
4. The growth rate initially increases with increasing pump-field amplitude, reaches a maximum at an optimum pump intensity, and subsequently decreases at higher pump fields due to nonlinear saturation effects. Therefore, an optimum pump-field amplitude exists for maximizing the modulational amplification.

Overall, the present investigation demonstrates that weakly-polar, heavily doped magnetoactive III–V semiconductor crystals constitute highly promising media for realizing efficient PL–TOP interaction-based modulational instability and amplification. The results presented here provide useful guidelines for the design and optimization of nonlinear semiconductor-based photonic and optoelectronic devices operating through modulational instability mechanisms.

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Authors' contributions

The authors read and approved the final manuscript.

Conflicts of interest

The authors declares no conflict of interest.

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Data availability

No new data were created.

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